Example 1:

\[ S = \{ q(f(X), X), q(f(h(T)), g(U)) \} \]

\[ \theta_0 = \epsilon \quad S\theta_0 = S \quad D_0 = \{ X, h(T) \} \]

\[ \theta_1 = \theta_0 \circ [X \mapsto h(T)] = [X \mapsto h(T)] \]

\[ S\theta_1 = \{ q(f(h(T)), h(T)), q(f(h(T)), g(U)) \} \]

\[ D_1 = \{ h(T), g(U) \} \]

\[ D_1 \text{ does not contain a variable, so we halt and report non-unifiability.} \]

Example 2:

\[ S = \{ p(X, f(X)), p(Y, Y) \} \]

\[ \theta_0 = \epsilon \quad D_0 = \{ X, Y \} \]

\[ \theta_1 = [X \mapsto Y] \]

\[ S\theta_1 = \{ p(Y, f(Y)), p(Y, Y) \} \]

\[ D_1 = \{ f(Y), Y \} \]

\[ D_1 \text{ contains a variable } Y, \text{ but this occurs in } f(Y), \text{ so we halt and report non-unifiability.} \]

Example 3:

\[ S = \{ q(X, f(X), g(a)), q(v, f(g(T)), V) \} \]

\[ \theta_0 = \epsilon \quad D_0 = \{ X, V \} \]

\[ \theta_1 = [X \mapsto V] \]

\[ S\theta_1 = \{ q(V, f(V), g(a)), q(V, f(g(T)), V) \} \]
\( D_1 = \{ V, g(T) \} \)

\[
\begin{align*}
\theta_2 &= \theta_1 \circ [V \mapsto g(T)] \\
&= [X \mapsto V] \circ [V \mapsto g(T)] \\
&= [X \mapsto g(T), V \mapsto g(T)]
\end{align*}
\]

\[ S\theta_2 = \{ q(g(T), f(g(T)), g(a)), q(g(T), f(g(T)), g(T)) \} \]

\( D_2 = \{ a, T \} \)

\[
\begin{align*}
\theta_3 &= \theta_2 \circ [T \mapsto a] \\
&= [X \mapsto g(T), V \mapsto g(T)] \circ [T \mapsto a] \\
&= [X \mapsto g(a), V \mapsto g(a), T \mapsto a]
\end{align*}
\]

\[ S\theta_2 = \{ q(g(a), f(g(a)), g(a)), q(g(a), f(g(a)), g(a)) \} \]

\[ = \{ q(g(a), f(g(a)), g(a)) \} \]

which is a singleton, so we halt and output the most general unifier \([X \mapsto g(a), V \mapsto g(a), T \mapsto a]\)