COMP 2411 Midterm Exam, Session 1, 2000

Answer all questions. Select exactly one item from each question. Incorrect answers will not be given negative marks.

Note: Read each question carefully. Some questions ask you to select which choice is false!

1. Consider the formulae (1) \( p \to (\neg q \lor r) \) and (2) \( q \equiv (r \land p) \). On a line of the truth table where \( p \) is true and \( q \) and \( r \) are both false,
   
   (a) both (1) and (2) are true.
   (b) (1) is true and (2) is false.
   (c) (1) is false and (2) is true.
   (d) both (1) and (2) are false.

2. Which of the following sentence structures in English does not correspond to the formula \( p \to q \)?
   
   (a) If \( p \) then \( q \).
   (b) \( q \) if \( p \).
   (c) \( p \) only if \( q \).
   (d) \( q \) only if \( p \).

3. A formula of propositional logic is valid if
   
   (a) it is true on some line of the truth table.
   (b) it is true on every line of the truth table.
   (c) it is false on some line of the truth table.
   (d) it is false on every line of the truth table.

4. Which of the following is a valid logical equivalence?
   
   (a) \( p \land (q \lor r) \equiv (p \lor q) \land (p \lor r) \)
   (b) \( p \to (q \to r) \equiv (p \to q) \to r \)
   (c) \( p \to q \equiv \neg p \lor q \)
   (d) \( p \equiv (q \land r) \equiv (p \equiv q) \land (p \equiv r) \)
5. Consider the following two arguments: (1) \( p \rightarrow (q \rightarrow r), \ q \models p \rightarrow r \) and (2) \( p \rightarrow (q \rightarrow r), \ \neg r \models \neg q \rightarrow \neg p \). Which of the following is true?

(a) Both (1) and (2) are valid.
(b) (1) is valid but (2) is not.
(c) (2) is valid but (1) is not.
(d) Both (1) and (2) are invalid.

6. Consider the following set of clauses.

\[
\begin{align*}
p & \rightarrow q \\
q \land r & \rightarrow p \\
s & \rightarrow r \\
\top & \rightarrow s \\
p & \rightarrow \bot
\end{align*}
\]

Which of the following is true?

(a) The algorithm HORN cannot be applied to this set of clauses because they are not all Horn clauses.
(b) The algorithm HORN can be applied to this set of clauses, but it does not terminate.
(c) The algorithm HORN can be applied to this set of clauses, and it returns "satisfiable".
(d) The algorithm HORN can be applied to this set of clauses, and it returns "unsatisfiable".

7. Which of the following is NOT known to be true?

(a) There exists an efficient algorithm for determining if a set of Horn clauses is satisfiable.
(b) There exists an efficient algorithm for determining if a set of Horn clauses is unsatisfiable.
(c) There exists an efficient algorithm for determining if the conjunction of a set of clauses is satisfiable.
(d) There exists an efficient algorithm for determining if the conjunction of a set of clauses is valid.

8. The result of converting the formula \( \neg((p \lor q) \land (r \land s)) \) to conjunctive normal form

(a) contains exactly one clause.
(b) contains exactly two clauses.
(c) contains exactly three clauses.
(d) contains more than three clauses.
9. A conjunctive normal form formula is valid if
   (a) it is possible to derive $\bot$ by resolution from the set of clauses in the formula.
   (b) it is not possible to derive $\bot$ by resolution from the set of clauses in the formula.
   (c) some clause in the formula contains a pair of complementary literals.
   (d) every clause in the formula contains a pair of complementary literals.

10. Consider the clauses
    
    $p \lor q \lor \neg r$
    $\neg p \lor r$
    $\neg q \lor \neg r$

    Which of the following clauses cannot be produced from these clauses by a single step of resolution?
    (a) $q$
    (b) $\neg r \lor p \lor \neg r$
    (c) $r \lor q \lor \neg r$
    (d) $\neg p \lor \neg q$

11. The formula $p \lor q \lor \neg r \lor \neg s$
    (a) is a definite Horn clause.
    (b) is not a clause.
    (c) is Horn clause but not a definite Horn clause.
    (d) is a clause but not a Horn clause.

12. Consider the following set of clauses:
    
    $p \rightarrow q$
    $q \land r \rightarrow p$
    $r \land s \rightarrow p$
    $\top \rightarrow r$
    $p \rightarrow \bot$

    Construct the proof tree produced by SLD resolution with leftmost literal selection and top to bottom choice of clause. Which of the following statements about this tree is false?
    (a) The tree has an infinite branch.
    (b) The tree has infinitely many branches.
    (c) A depth first search of this tree will terminate.
    (d) The tree does not contain branch with $\bot$ at the leaf.
13. Consider the following incomplete natural deduction proof.

1:  p \rightarrow (q \rightarrow r) \quad \text{Premise}
2:  q \quad \text{Premise}
3:  r \rightarrow t \quad \text{Premise}
4:  ((p \rightarrow t) \land r) \rightarrow s \quad \text{Premise}

5:  \ldots \quad \text{Assumption}
6:  q \rightarrow r \quad \ldots
7:  r \quad 2, 6, \quad \rightarrow e
8:  t \quad \ldots

9:  p \rightarrow t \quad \ldots
10:  (p \rightarrow t) \land r \quad \ldots
11:  s \quad 4, 10, \quad \rightarrow e

Which of the following assertions is true?

(a) This cannot be completed to make up a correct proof, because nothing can be found to fill the dots at step 5 that will make step 9 correct.
(b) This cannot be completed to make up a correct proof, because step 10 cannot be justified.
(c) If we put a p at step 5 then we can fill in all the missing justifications so as to make up a correct proof.
(d) The justification for step 11 is incorrect.

14. Suppose that $\phi_1, \ldots, \phi_n \models \psi$. Which of the following is guaranteed to be true?

(a) Provided $\psi$ is a clause, we can convert $\phi_1, \ldots, \phi_n$ into a set of clauses, and use resolution to prove $\psi$.
(b) If we convert $\phi_1, \ldots, \phi_n$ and $\psi$ into a set of clauses, we can use resolution to prove $\bot$.
(c) If we convert $\phi_1, \ldots, \phi_n$ and $\neg \psi$ into a set of clauses, we can use resolution to prove $\bot$.
(d) We can convert $\phi_1, \ldots, \phi_n$ and $\neg \psi$ into a set of Horn clauses, and use the algorithm HORN to prove that this set is unsatisfiable.

15. Consider the following formula:

$$p(X,Y) \rightarrow \forall Z(q(X,Z) \land R(Y,Z) \rightarrow \exists Y(P(X,Y)))$$

(a) This formula has exactly one free occurrence of $X$.
(b) This formula has exactly two free occurrences of $X$.
(c) All the occurrences of $X$ in this formula are free.
(d) This formula is a sentence.
16. Consider the chess position depicted on the final page of this exam. Suppose we represent this position by a structure in which the universe is the set of all pieces and squares in this position. Define

- \( \text{pawn}(X) \) to be true just when \( X \) is a pawn, and
- \( \text{adjacent}(X, Y) \) to be true when \( X \) and \( Y \) are two distinct pieces that are on adjacent squares.

Which of the following formulae is true in this structure?

(a) \( \forall X (\text{pawn}(X) \rightarrow \exists Y (\text{pawn}(Y) \land \text{adjacent}(X, Y))) \)
(b) \( \exists X (\text{pawn}(X) \land \forall Y (\text{pawn}(Y) \rightarrow \text{adjacent}(X, Y))) \)
(c) \( \forall X (\text{pawn}(X) \rightarrow \forall Y (\text{pawn}(Y) \land \text{adjacent}(X, Y))) \)
(d) \( \forall X (\text{pawn}(X) \land \exists Y (\text{pawn}(Y) \land \text{adjacent}(X, Y))) \)

17. Consider the same structure as in the previous question. Which of the following formulae is true in this structure?

(a) \( \exists X \exists Y \exists Z (\text{pawn}(X) \land \text{pawn}(Y) \land \text{pawn}(Z) \land \text{adjacent}(X, Y) \land \text{adjacent}(Y, Z)) \)
(b) \( \exists X \exists Y (\neg \text{pawn}(X) \land \neg \text{pawn}(Y) \land \text{adjacent}(X, Y)) \)
(c) \( \forall X \forall Y ((\text{pawn}(X) \land \text{pawn}(Y)) \rightarrow \text{adjacent}(X, Y)) \)
(d) \( \forall X \forall Y ((\text{pawn}(X) \land \text{pawn}(Y)) \rightarrow \neg \text{adjacent}(X, Y)) \)

18. Which of the following formulas best expresses “As a general rule of chess, if two rooks of opposite colour are on adjacent squares then they can take each other.”

(a) \( \exists X \exists Y (\text{rook}(X) \land \text{rook}(Y) \land \text{adjacent}(X, Y) \land \neg \text{colour}(X) = \text{colour}(Y)) \rightarrow (\text{can} \rightarrow \text{take}(X, Y) \land \text{can} \rightarrow \text{take}(Y, X)) \)
(b) \( \exists X \exists Y ((\text{rook}(X) \land \text{rook}(Y) \land \text{adjacent}(X, Y) \land \neg \text{colour}(X) = \text{colour}(Y)) \rightarrow [\text{can} \rightarrow \text{take}(X, Y) \land \text{can} \rightarrow \text{take}(Y, X)]) \)
(c) \( \forall X \forall Y ((\text{rook}(X) \land \text{rook}(Y) \land \text{adjacent}(X, Y) \land \neg \text{colour}(X) = \text{colour}(Y)) \rightarrow [\text{can} \rightarrow \text{take}(X, Y) \land \text{can} \rightarrow \text{take}(Y, X)]) \)
(d) \( \forall X \forall Y (\text{rook}(X) \land \text{rook}(Y) \land \text{adjacent}(X, Y) \land [\neg \text{colour}(X) = \text{colour}(Y)) \rightarrow (\text{can} \rightarrow \text{take}(X, Y) \land \text{can} \rightarrow \text{take}(Y, X))] \)
19. Suppose that \( f \) is a function symbol of arity 2, \( g \) is a function symbol of arity one, \( c \) is a constant symbol and \( X \) is a variable. Let \( \mathcal{M} \) be a structure with

- universe \( A^\mathcal{M} \) equal to the set of natural numbers \( \mathbb{N} \),
- \( c^\mathcal{M} = 3 \),
- \( f^\mathcal{M}(m,n) = m + n \) for all \( m,n \in \mathbb{N} \),
- \( g^\mathcal{M}(n) = n + 1 \) for all \( n \in \mathbb{N} \).

and let \( \sigma \) be an environment with \( \sigma(X) = 2 \). Then

(a) \( \mathcal{M} \models_\sigma f(g(X), c) = c \)
(b) \( \mathcal{M} \models_\sigma f(g(X), c) = g(c) \)
(c) \( \mathcal{M} \models_\sigma f(g(X), c) = f(c, c) \)
(d) \( \mathcal{M} \models_\sigma f(g(X), c) = f(g(c), c) \)

20. Using the convention that variables are in upper case and constants in lower case, which of the following formulas of predicate logic is a sentence?

(a) \( \forall X (f(Z) = c \longrightarrow \exists Y (p(X, Y))) \)
(b) \( \forall X (f(X) = c \longrightarrow \exists Y (p(Z, Y))) \)
(c) \( \forall X (f(X) = c \longrightarrow \exists Y (p(Z, Y))) \)
(d) \( \forall X (f(c) = c \longrightarrow \exists Y (p(X, Y))) \)