COMP2411 Tutorial 4

1. Convert the following formulas to conjunctive normal form. Which of them are valid?
   (a) \((P \land Q) \rightarrow R\)
   (b) \(((P \land Q) \rightarrow R) \lor (\neg R \land P)\)
   (c) \(((P \land Q) \rightarrow R) \lor (R \rightarrow (P \leftrightarrow Q))\)

2. Use resolution to prove the following arguments are valid:
   (a) Frank took the Ford or the Toyota. If Frank took the Ford he will be late. Frank is not late. Thus, Frank took the Toyota.
   (b) If Phillipine Airlines flies to Hong Kong then if Qantas does not fly there, its profit will be reduced. Thus, if Qantas does not fly to Hong Kong, then if Phillipine Airlines flies to Hong Kong then the Qantas profit will be reduced.
   (c) The alarm should not sound unless the system has been armed or there is a fire. If the system has been armed and a door is disturbed, the alarm should ring. Irrespective of whether the system has been armed, the alarm should go off when there is a fire. Thus, if the alarm is ringing and there is no fire, then the system must have been armed.

3. Use resolution to prove that a graph with three vertices, with an edge between every two vertices, is not two-colourable.

Optional Advanced Questions

1. We mentioned in the lecture that resolution is sound and complete for derivation of \(\bot\). Is resolution sound and complete for derivation of clauses? I.e. is it the case that if \(S\) is a set of clauses and \(c\) a clause, then \(S \models c\) if and only if \(S \vdash_{\text{Res}} c\)?

2. A programming problem: Implement a program that reads a set of clauses from a file and tries to derive \(\bot\) using resolution. Try to ensure that it will halt, given enough time.